Area of a Smile

The region between the curves $y = \frac{1}{2}x^2 - \frac{1}{2}$ and $y = x^4 - 1$ is smile shaped. Find the area of that region.

9/8/25

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$$\frac{1}{2}x^2 - \frac{1}{2} = x^4 - 1$$

$$x^4 - \frac{1}{2}x^2 - \frac{1}{2} = 0$$

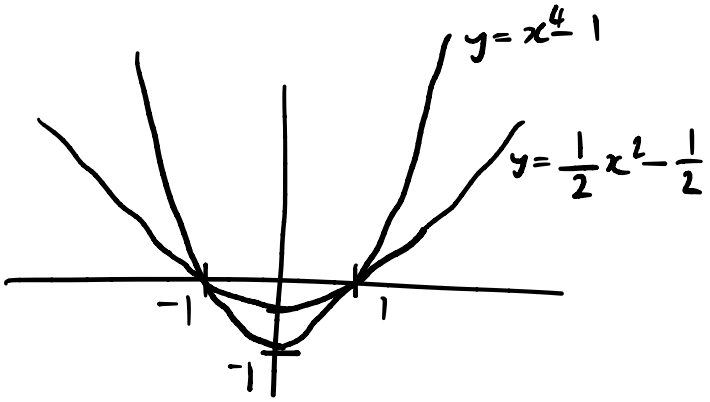
$$(x-1)\left(x^3 + x^2 + \frac{1}{2}x + \frac{1}{2}\right) = 0$$

$$(x-1)(x+1)\left(x^2 + \frac{1}{2}\right) = 0$$

1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
		1	1	$\frac{1}{2}$	$\frac{1}{2}$
	1	1	$\frac{1}{2}$	$\frac{1}{2}$	0

$$\left(x^3 + x^2 + \frac{1}{2}x + \frac{1}{2}\right)$$

-1	1	1	$\frac{1}{2}$	$\frac{1}{2}$
		-1	0	$-\frac{1}{2}$
	1	0	$\frac{1}{2}$	0



$$\text{Area} = \int_{-1}^{1} \frac{1}{2}x^2 - \frac{1}{2} - (x^4 - 1)\,dx$$

$$= \frac{x^3}{6} - \frac{1}{2}x - \frac{x^5}{5} + x \Big|_{-1}^{1}$$

$$= \left(\frac{1}{6} - \frac{3}{6} - \frac{1}{5} + 1\right) - \left(-\frac{1}{6} + \frac{3}{6} + \frac{1}{5} - 1\right)$$

$$= \left(\frac{20}{30} - \frac{6}{30}\right) - \left(-\frac{20}{30} + \frac{6}{30}\right)$$

$$= \frac{14}{30} + \frac{14}{30} = \frac{28}{30} = \frac{14}{15}$$